CONJUGATE PROBLEM OF THE FREE CONVECTION IN A VERTICAL CHANNEL, TAKING INTO ACCOUNT THE RHELOGICAL TEMPERATURE DEPENDENCE

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The conjugate problem of natural convection of a power-law liquid in a vertical channel, taking account of the temperature-dependent consistency index of the liquid, is investigated. The internal heat sources are also taken into account. The influence of the parameters of the problem on the convection and heat flux is also discussed in detail.

1. The general formulation of the conjugate problem of natural convection of non-Newtonian liquid with a rheological power law in a plane vertical channel was given in [1]. Consider the case where the temperature dependence of the consistency index k of the liquid is

$$k(T) = A\exp(-bT/T_0),$$

where A, b are constants. The constants of the internal heat sources q in the liquid and q_1, q_2 in the walls are also taken into account.

For a completely developed one-dimensional process, the initial system of equations is written in dimensionless form as follows (the tilde is omitted; Fig. 1)

$$\frac{d}{dx}\left[\exp(-\alpha\theta)\left|\frac{dv}{dx}\right|^{n-1}\frac{dv}{dx}\right]+\theta=\frac{dp}{dy}=F, \tag{1}$$

$$\frac{d^2\theta}{dx^2}+S=0 \tag{2}$$

with the boundary conditions

$$\frac{d^2\theta_1}{dx^2}+S_1=0, -\left(1+\frac{h}{l}\right)\leqslant x\leqslant -1;$$

$$\frac{d^2\theta_1}{dx^2}+S_2=0,\ 1\leqslant x\leqslant\left(1+\frac{h}{l}\right);$$

$$x=\pm 1:\ v=0;\ \theta=\theta_1;\ \frac{d\theta}{dx}=\frac{\lambda_1}{\lambda}\frac{d\theta_1}{dx};$$

$$x=\pm\left(1+\frac{h}{l}\right):\ \theta_1=\pm 1;\ \int\limits_{-1}^{+1} v\,dx=0.$$

The scales of distance, velocity, pressure, and temperature are, respectively: l, $V=[\rho g\beta\Delta T l^{n+1}/A\exp(-b)]^{\frac{1}{n}}$; $P=\rho g\beta\Delta T$; ΔT. Here $2\Delta T$ is the temperature difference between the walls; and

$$\tilde{x}=\frac{x}{l};\ \tilde{v}=\frac{v}{V};\ \tilde{p}=\frac{p}{P};\ \theta=\frac{T-T_0}{\Delta T};\ \theta_1=\frac{T_1-T_0}{\Delta T};$$

$$\alpha=b\frac{\Delta T}{T_0};\ S=\frac{ql^2}{\lambda\Delta T};\ S_1=\frac{q_1l^2}{\lambda_1\Delta T};\ S_2=\frac{q_2l^2}{\lambda_1\Delta T};\ F=\frac{C}{\rho g\beta\Delta T},$$

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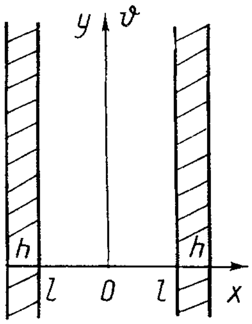

Fig. 1. Scheme of problem.

where C is the constant of variable separation.

Note that the boundary conditions for the temperature may be written in a different form [2]

$$x=-1:\ \frac{d\theta}{dx}=\frac{1+\theta}{\psi}-\frac{S_1}{2\psi}\left(\frac{h}{l}\right)^2;\quad x=+1:\ \frac{d\theta}{dx}=\frac{1-\theta}{\psi}+\frac{S_2}{2\psi}\left(\frac{h}{l}\right)^2. \tag{3}$$

The conjugacy parameter $\psi = \lambda h/\lambda_1 \ell$ characterizes the ratio of thermal conductivities and thicknesses in the liquid—wall system.

With the boundary conditions in Eq. (3), Eq. (2) for the temperature field has the solution

$$\theta(x)=E+2Bx-3Dx^2. \tag{4}$$

Substituting Eq. (4) into Eq. (1) and integrating, the velocity distribution is obtained

$$v(x)=\int_{-1}^{x} W(t)\,dt. \tag{5}$$

Here

$$W(t)=|D_1+(F-E)t-Bt^2+Dt^3|^{\frac{1}{n}}\operatorname{sign}(D_1+(F-E)t-$$
$$-Bt^2+Dt^3)\exp\left[\frac{\alpha}{n}(E+2Bt-3Dt^2)\right];$$
$$E=\left(\frac{h}{l}\right)^2\frac{S_2+S_1}{\psi}+\frac{S}{2}(1+2\psi);$$
$$2B=\frac{4+\left(\frac{h}{l}\right)^2(S_2-S_1)}{4(1+\psi)};\ 3D=\frac{S}{2}.$$

The parameter α characterizes the degree of temperature dependence of k. When $\alpha = 0$, the case considered in [1] is obtained. Note that in the expression for W(t) there are two unknown constants F and D_1, which are determined from the conditions

$$v(1)=0; \tag{6}$$

$$\int_{-1}^{+1} v(x)\,dx=0. \tag{7}$$

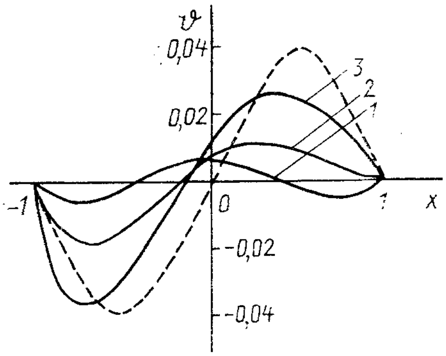

Fig. 2. Velocity profiles when $\alpha = 0$, $n = 0.6$, $D = 0.6$ for $B = 0$ (1); 0.3 (2); 0.6 (3); dashed curve: velocity v_1 for $n = 1$; $D = 0$, $B = 0$, $v_1 = 40v$.

The heat flux is now calculated

$$Q = \rho c_p \int_{-l}^{+l} vT dx,$$

or in dimensionless form

$$Q = \int_{-1}^{+1} v\theta dx. \tag{8}$$

Substituting θ from Eq. (4) into Eq. (8) and integrating by parts gives the following result, taking into account that $v(+1) = v(-1) = 0$

$$Q = -B \int_{-1}^{+1} x^2 W(x)\, dx + D \int_{-1}^{+1} x^3 W(x) dx. \tag{9}$$

2. To calculate $v(x)$ and Q, the unknowns F and D_1 must be determined. From Eq. (6)

$$\int_{-1}^{+1} W(x)\, dx = 0. \tag{10}$$

For Eq. (7), calculation of the double integral is replaced by integration by parts, to give

$$\int_{-1}^{+1} xW(x)\, dx = 0. \tag{11}$$

The integrals in Eqs. (10) and (11) are calculated approximately by the Gaussian formula

$$\sum_{i=1}^{M} a_i W(x_i, D_1, F) = 0; \tag{12}$$

$$\sum_{i=1}^{M} a_i x_i W(x_i, D_1, F) = 0, \tag{13}$$

where M, a_i, x_i are the number of points, the load, and the coordinate, respectively.

Newton's method is used to solve Eqs. (12) and (13), which are linear algebraic equations with two unknowns D_1 and F. The initial values for D_1 and F are: $D_1^0 = B/3$; $F^0 = E - 3D/5$ (accurate values when $n = 1$).

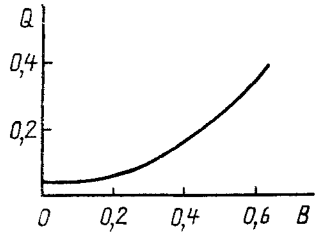

Fig. 3. Distribution of heat flux as a function of B when $\alpha = 0$, $n = 0.6$, $D = 0.6$.

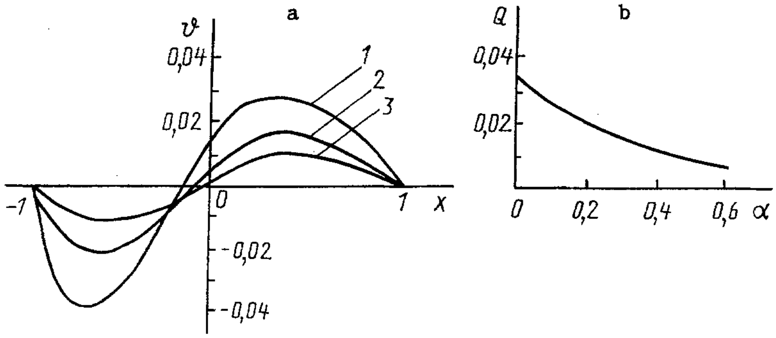

Fig. 4. Velocity (a) and heat flux (b) profiles when $n = 0.6$, $E = 0$, $D = B = 0.6$: 1) $\alpha = 0$; 2) 0.3; 3) 0.6.

3. The velocity distribution and heat flux are now analyzed in more detail. Consider the case when $\alpha = 0$.

Analysis of the expressions for the velocity and heat flux — Eqs. (5) and (9) — and the conditions in Eqs. (10) and (11) shows that $v(x)$ and Q are related to the parameters S, S_1, S_2, ψ, h/ℓ only through D and B and do not depend on E. Physically, this means that, if the temperature increases or decreases by the same amount, $v(x)$ and Q are unchanged.

If there is no heat source in the liquid, i.e., $D = 0$, the velocity distribution is symmetric with respect to the initial coordinate; $v(x)$ is an odd function of x. The liquid in the channel moves in two opposite fluxes: an ascending flux at the right-hand wall (with higher temperature) and a descending flux at the left-hand wall. In this case, the velocity takes the form

$$v(x) = \int_0^x |D_1 - Bt^2|^{\frac{1}{n}} \operatorname{sign}(D_1 - Bt^2)\, dt =$$

$$= B^{\frac{1}{n}} \int_0^x |D_1^* - t^2|^{\frac{1}{n}} \operatorname{sign}(D_1^* - t^2)\, dt = B^{\frac{1}{n}} \bar{v}(x, n).$$

Here $\bar{v}(x, n)$ depends only on n, and is an odd function of x. Thus, the parameters S_1. S_2, ψ, h/ℓ have no influence on the velocity profile, apart from changing its scale. The convection intensity increases with increases in B and vice versa.

Analogously, the expression for the heat flux is

$$Q = B^{1+\frac{1}{n}} \bar{Q}(n),$$

where $\bar{Q}(n)$ depends only on n.

If $B = 0$ or $(h/\ell)^2 (S_2 - S_1) = 4$, the velocity distribution is symmetric with respect to the axis oy; $v(x)$ is an even function of x. Then the liquid moves in three fluxes: two descending fluxes at the wall and an ascending flux at the center.

The expression for the velocity is

$$v(x) = \int_{-1}^{x} |Dt^3 + (F-E)t|^{\frac{1}{n}} \operatorname{sign}(Dt^3 + (F-E)t)\, dt =$$

$$= D^{\frac{1}{n}} \int_{-1}^{x} |t^3 + (F-E)^* t|^{\frac{1}{n}} \operatorname{sign}(t^3 + (F-E)^* t)\, dt = D^{\frac{1}{n}} \bar{\bar{v}}(x,\ n),$$

where $\bar{\bar{v}}(x)$ is an even function of x (fig. 2). As above, the velocity distribution does not depend on S, S_1, S_2, ψ, h/ℓ in this case. The convection increases with increase in D — i.e., with increase in S — and vice versa.

For the heat flux

$$Q = D^{1+\frac{1}{n}} \bar{\bar{Q}}(n).$$

When B $\neq$ 0

$$v(x) = B^{\frac{1}{n}} v^* \left(\frac{D}{B},\ n,\ x \right);\quad Q = B^{1+\frac{1}{n}} Q^* \left(\frac{D}{B},\ n \right).$$

When D/B decreases (for example, on account of decrease in S or ψ), the descending flux of liquid at the left-hand wall expands and that at the right-hand wall contracts, so that their sum is equal to the ascending flux in the center. When D/B decreases to a certain value, the liquid flux at the right-hand wall disappears and only two fluxes remain in the channel: a left-hand descending flux and a right-hand ascending flux (Fig. 2). This means that, when the heat source S and the conjugacy parameter ψ are slight, the problem reduces to the well-known case of natural convection in a vertical channel with constant but different temperatures.

The heat flux is shown in Fig. 3 as a function of B at fixed n, D. With increase in B — i.e., decrease in ψ — the heat flux entrained by the moving liquid along the wall increases.

4. Consider the case when $\alpha \neq 0$. In contrast to the case when α = 0, v(x) and Q depend on the three parameters D, B, E here

$$v(x) = \exp\left(\frac{\alpha}{n} E \right) v^0(x,\ B,\ D,\ \alpha),$$

$$Q = \exp\left(\frac{\alpha}{n} E \right) Q^0(B,\ D,\ \alpha).$$

It is evident that E has no influence on the form of the velocity profile.

If B = 0, the velocity distribution is symmetric with respect to oy, as in the previous case

$$v(x) = \int_{0}^{x} |(F-E)t + Dt^3|^{\frac{1}{n}} \operatorname{sign}((F-E)t + Dt^3) \times \exp[-\alpha(E - 3Dt^2)]\, dt + D_2.$$

In general, the appearance of an exponential term in the expressions for v(x) and Q means that the intensity of the convective flow and the heat flux in the channel decreases (Fig. 4), while v(x) is only an even function when D = 0. In addition, the velocity distribution depends on D and B and not on their ratio, as in the case when α = 0.

For a clearer idea of the variation in velocity and heat flux, profiles of v and Q are plotted in Fig. 4 for fixed n, D, B, E and various α. It is evident that the influence of the temperature-dependent consistency index k of the liquid on the velocity and heat flux is significant. With increase in α, the convection and heat flux decrease.

LITERATURE CITED

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DRIFT OF A RAREFIED GAS IN A PLANAR CHANNEL UNDER THE ACTION OF MONOCHROMATIC RADIATION

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A theoretical study is performed of light-induced drift of a rarefied gas in a planar channel. The problem is solved using linearized kinetic equations for a two-level particle model in the "weak field" approximation. Analytical expressions are obtained for the drift, averaged over channel section in almost free-molecular ($Kn \gg 1$) and viscous with slippage ($Kn \ll 1$) flow regimes. Various mechanisms for this drift are analyzed. Numerical estimates of drift velocity are presented.

The phenomenon of light-induced drift involves [1] particles absorbing radiation in the form of a travelling wave while located in a mixture with a buffer gas, and taking on a directed motion (drift). The drift may occur in the direction of radiation propagation or opposite thereto. It has been established that this drift phenomenon is realized in cases typical of nonlinear optics and spectroscopy problems, and is inherent to several classes of particles: atoms, molecules, and ions.

The great majority of studies of light-induced drift have investigated the phenomenon in an infinite gas. At the same time it is clear that light-induced drift is also possible in the case where the role of the buffer gas is played by an interphase surface, which reflects excited and unexcited particles in different manners. The factor of gas—surface interaction becomes one of the dominant ones in drift motion in capillary-porous media.

In [2] gas drift was studied in a vessel, the dimensions of which were large in comparison to the atomic free path length. The model used in [2] of strong collisions with a single Maxwell collision frequency can be considered only as a first approximation for description of light-induced drift. In particular, it does not consider the collision mechanism of light-induced drift development related to difference in the frequency of collisions of excited and unexcited atoms among themselves [3, 4]. Moreover, to calculate the velocity of light-induced slippage, [2] considered the spatially homogeneous case, although it is precisely in the Knudsen layer that the macroparameters (and thus, the distribution function) experience their greatest changes.

In [5] light-induced drift was studied in a planar channel on the basis of specific model equations [6], according to which each act of interatomic collision leads to extinction of an excitation, and the saturation parameter is independent of particle velocity in the resonance region. The latter is possible only in the case of detuning of the radiation frequency from resonance Ω and a Doppler shift kv. In [6] the accommodation mechanism of light-induced drift was not considered.

In the present study light-induced drift in a plannar channel will be described by second order model kinetic equations, which in contrast to the strong collision model include as macroparameters the gas velocity and the stress tensor, as well as considering three types of interparticle interactions. The problem is solved by the variation method, which permits achievement of sufficiently precise results over the entire Knudsen number (Kn) range, and in addition leads to simple analytical expressions for the light-induced drift in the two limiting cases $Kn \ll 1$ and $Kn \gg 1$. This permits a clear analysis of the contribution of both the collision and the accommodation mechanisms to gas drift at various Kn.

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